Stochastic entrainment of optical power dropouts

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We show that the natural pulsed behavior, in the form of sudden power dropouts, exhibited by semiconductor lasers subject to optical feedback can be entrained by the joint action of external noise and weak periodic driving. These power dropouts, which in the absence of forcing do not occur periodically, acquire the periodicity of the harmonic driving for an optimal amount of external noise, in what constitutes a form of stochastic resonance. This phenomenon is analyzed by means of a generalized Lang-Kobayashi model with external nonwhite noise in the modulated pump current, in terms of both the temporal correlation and the amplitude of the noise.

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Self-pulsing is one of the most characteristic dynamical regimes exhibited by nonlinear systems. When this behavior is due to the existence of a limit cycle, the evolution of the system is periodic. In some other situations, however, nonperiodic trains of pulses can be observed. This is the case, for instance, of the dynamical evolution of excitable systems, which have a single stable fixed point, but where unavoidable fluctuations may take the system to large excursions in phase space, before returning to its steady state. Given the random nature of these noise-driven escape processes, the resulting pulse trains are not periodic.

Nonperiodic self-pulsations also occur in other kinds of autonomous nonlinear systems. An example can be found in the dynamics of semiconductor lasers with optical feedback which, close to the solitary-laser threshold and for weak to moderate feedback levels, exhibit sudden dropouts in the evolution of the emitted intensity, at frequencies much lower ($\sim$tens of megahertz) than those typical of the system ($\sim$ GHz) [1]. In the standard case of a constant pumping current, and with rare exceptions [2,3], the resulting pulse train is irregular, a characteristic for which both deterministic [4,5] and stochastic [6] mechanisms have been proposed.

Interest has arisen recently in controlling the irregular pulsed dynamics mentioned above. In the case of excitable systems, for instance, it has been seen that an externally applied noise is able to induce and enhance the periodic response of the system, in what has been called *coherence resonance* [7], a phenomenon that occurs even for chaotic systems [8]. A more straightforward technique, however, is to use an external periodic forcing. Such a procedure has already been implemented, for instance, in semiconductor lasers with optical feedback, leading to entrainment of the power dropouts by a large enough harmonic modulation of the pumping current of the laser, provided the modulation frequency lies within an appropriate range of values [9]. Recent experimental work [10] has analyzed the influence of external noise on this entrainment.

In the present paper we show that a very weak modulation can also lead to entrainment, provided a suitable amount of external noise is added to the injection current. This behavior, which presents the typical hallmarks of *stochastic resonance* [11], can also be understood as a noise-based, nonfeedback scheme for controlling chaos (given that the dropout events can be interpreted as the result of chaotic itinerancy with a drift [12]). It should be noted that recent experimental studies of the process of escape from the high-gain mode of a stable semiconductor laser with feedback in the presence of external noise and modulation have shown no trace of stochastic resonance [13]. Our situation is different, since the laser is not operating in the high-gain mode.

Our analysis will be performed in the framework of the well-known Lang-Kobayashi (LK) model, which describes the dynamical behavior of a single-longitudinal-mode semiconductor laser subject to optical feedback, considering only one single reflection from the external mirror. The LK model describes the temporal evolution of the slowly varying complex envelope of the electric field $E(t)$ and the excess carrier number $N(t)$. In dimensionless form the model reads [14]:

$$\frac{dE}{dt}=\frac{1+i\alpha}{2}[G(E,N)-\gamma]E(t)+\kappa e^{-i\omega\tau_f}E(t-\tau_f)+\sqrt{2\beta N}\zeta(t),$$

$$\frac{dN}{dt}=\gamma_e[C(t)N_{\rm th}-N(t)]-G(E,N)|E(t)|^2, \qquad (1)$$

where γ and γ_e are the inverse lifetimes of photons and carriers, respectively, α is the linewidth enhancement factor, and ω is the free-running-laser frequency. The pumping term $C(t)$ has the form $C(t)=C_0[1+\xi(t)+A\sin(\Omega t)]$, where C_0 is the bias pumping rate (directly related to the dc driving current; $C=1$ is the solitary-laser threshold), and which is affected by a random term [represented by $\xi(t)$] and a harmonic driving of amplitude A and frequency Ω. The last term in the electric-field equation represents spontaneous emission fluctuations, with $\zeta(t)$ given by a Gaussian white noise of zero mean and unity intensity, and β measuring the internal noise strength. This noise cannot be controlled externally, thus it is not suitable for a systematic analysis and will not be used here as a control parameter.

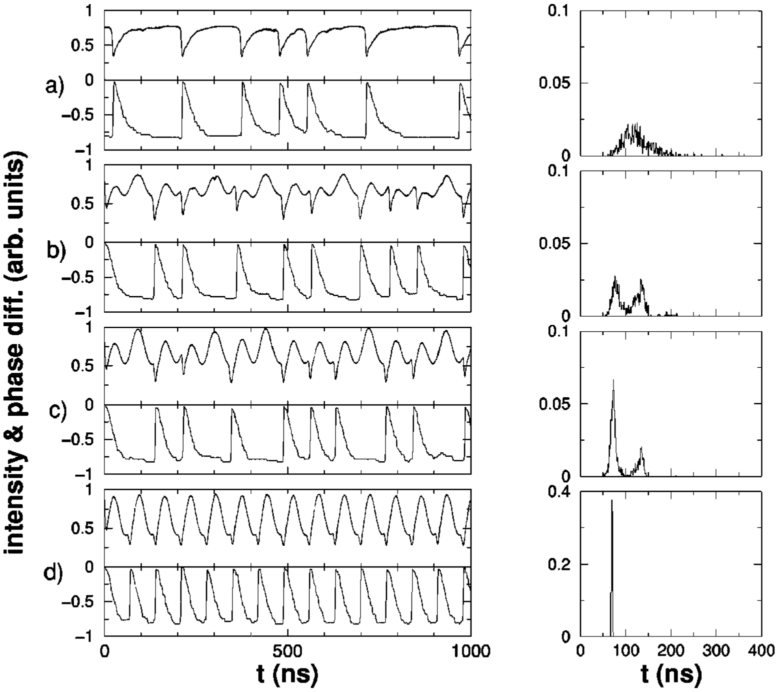

FIG. 1. Obtaining entrainment by modulation. Time evolution of the laser intensity (left column, upper plots), the phase difference $\eta(t)$ (left column, lower plots), and the corresponding PDFs of the dropout periods (right column) for increasing values of modulation amplitude and in the absence of noise. (a) $A=0.00$; (b) $A=0.03$; (c) $A=0.05$; (d) $A=0.08$. Modulation period is set to $T=70$ ns. Other parameters of the model are $C_0=1.03$, $\gamma_e=6\times10^{-4}$ ps^{-1}, γ=0.158 ps^{-1}, $g=2.79\times10^{-9}$ ps^{-1}, $s=3\times10^{-7}$, α=4.0, $N_0=1.51\times10^8$, $\beta=5\times10^{-10}$ ps^{-1}, $\kappa=0.025$ps^{-1}, τ_f=2.4 ns, and $\omega\tau_f=2$.

The material-gain function $G(E,N)$ is given by

$$G(E,N)=\frac{g[N(t)-N_0]}{1+s|E(t)|^2}, \tag{2}$$

where g is the differential gain coefficient, N_0 is the carrier number in transparency, and s is the saturation coefficient. The threshold carrier number is $N_{\text{th}}=\gamma/g+N_0$. Finally the optical feedback term is described by two parameters: the feedback strength κ and the external round-trip time τ_f.

As mentioned above, and in spite of the relatively low frequencies of the dropout events, this system is characterized by a very fast dynamics (with characteristic times of the order of picoseconds) [12,15]. Therefore, and due to bandwidth limitations of the electronics involved, any experimental implementation of electrical noise in the injection current will lead to an external noise with non-negligible correlation time. Hence, the noise term $\xi(t)$ in our model is taken to be a time-correlated Gaussian random process of the Ornstein-Uhlenbeck type, with zero mean and correlation

$$\langle \xi(t)\xi(t')\rangle=\frac{D}{\tau_c}e^{-|t-t'|/\tau_c}. \tag{3}$$

This external noise is characterized by two parameters, its intensity D and its correlation time τ_c. The variance of the noise is given by D/τ_c, and hence we will measure its amplitude as $\sigma=\sqrt{D/\tau_c}$.

When the laser is operating near threshold, under optical feedback and without external noise or modulation, low-frequency intensity drops, which are known as low-frequency fluctuations (LFF), appear at irregular times. As mentioned earlier, one method of entraining the intensity dropouts is by applying a modulation signal to the input current of the laser [9]. In Fig. 1 we present the time series of the output intensity, the phase difference (defined below) and the probability distribution function (PDF) of the time between dropouts for different modulation amplitudes. We initially feed the laser to be in the LFF regime [Fig. 1(a)] and then a weak sinus-modulated wave is added to the system through the pumping current. The modulation period ($T=70$ ns) is chosen within the range where the response to the harmonic signal is optimal [9]. As it can be seen, when the modulation amplitude is increased, intensity dropouts begin to occur at periods multiples of the modulation [Figs. 1(b) and 1(c)]. If the amplitude is further increased, we can force the system to follow the modulation period [Fig. 1(d)]. To observe the entrainment better, Fig. 1 also shows the phase difference between consecutive round trips $\eta(t)=\phi(t)-\phi(t-\tau_f)$, where $\phi(t)$ is the phase of the electric field defined as $E(t)=\sqrt{I}\exp(i\phi)$. Note the relation between the phase difference $\eta(t)$ and the intensity $I(t)$, where each intensity dropout corresponds to a pulse in the phase difference evolution. On the other hand, the intensity evolution has been numerically filtered at 100 MHz to simulate the response obtained by a slow photodetector, in order to qualitatively compare numerical simulations with experimental data. The PDFs of the time between dropouts, show how the period distribution between dropouts has a maximum at the modulation period when the amplitude is sufficiently large.

Another way of obtaining entrainment is based on the phenomenon of stochastic resonance (SR). In SR we initially have a nonlinear system with a weak external forcing. When noise is added to the system, and for an optimal range of noise amplitude, the periodic response of the system is enhanced. Applying the SR concept to our system, we modulate the laser with a weak external signal, while the laser is operating in the LFF regime. We will now show that we can force the intensity dropouts to follow the modulation signal just by applying the right amount of noise. External noise will be introduced into the system through the injection cur-

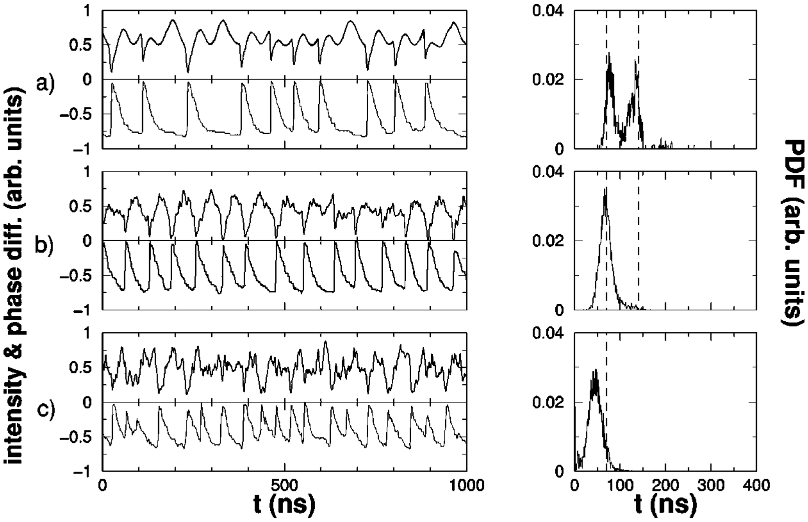

FIG. 2. Time evolution of the laser intensity (left column, upper plots), the phase difference $\eta(t)$ (left column, lower plots), and the corresponding PDFs of the dropout periods (right column) for increasing values of the external-noise strength. (a) $D=0.00$ ps, (b) $D=0.25$ ps, (c) $D=0.80$ ps. The noise correlation time is set to $\tau_c=240$ ps, the amplitude of the modulation is $A=0.03$, and its period is $T=70$ ns. Other parameters of the model are those of Fig. 1.

rent of the laser, as mentioned earlier. In Fig. 2 we show the intensity and phase difference time traces and the PDF for three different values of the noise amplitude ($\sigma=\sqrt{D/\tau_c}$). We initially set the modulation amplitude to 3% of the threshold current [$A=0.03$ corresponding to Fig. 1(b)] and the modulation period at $T=70$ ns. The noise correlation time is fixed to $\tau_c=240$ ps, since this value has been shown to be optimal for coherence resonance in the same system [16]. In the absence of noise [Fig. 2(a)], and with these parameters, the PDF presents two peaks at periods around 70 ns and 140 ns, which are multiples of the modulation period. When noise intensity is increased to intermediate values, $D=0.25$ ps, one can observe just one peak in the PDF, centered at ~ 70 ns, i.e., at the modulation period [Fig. 2(b)]. If noise intensity is further increased the system loses the regularity, and dropouts fall at much lower periods, where LFF are difficult to observe [Fig. 2(c)].

Hence Fig. 2 displays the main characteristics of SR, namely, that for a certain range of noise intensities the period of the LFF follows the modulation period, showing a peak in the PDF at ~ 70 ns. As a consequence, we can say that noise not only helps the system to follow a weak driving but also gives regularity to the dropout series. This enhancement of the LFF regularity is observed in Fig. 3, where the standard deviation of the normalized dropout periods is shown for different noise intensities. A minimum at noise intensity $D=0.25$ ps can be observed. This minimum implies a maximum regularity of the dropouts. But the main point of the stochastic entrainment is that the mean value of the dropout periods at the maximum regularity ($D=0.25$ ps) coincides with the modulation period, $T=70$ ns (see Fig. 3 upper trace). Thus, we observe that noise helps the system to respond to the external forcing when the system is not able by itself (due to the weakness of the modulation signal), a typical feature of SR.

In Fig. 4 we present the PDF obtained for increasing values of the noise intensity. When analyzing the evolution of the maximum values of the PDF in each plot we observe, for a noise intensity $D=0.0$ ps, two peaks located at periods T and $2T$. This means that modulation is not strong enough to force the system to follow its period, a fact that would be possible if modulation amplitude were increased [see Fig. 1(d)]. Although the system is not able to follow the modulation amplitude it is clear that the dynamics is dominated by the modulation, since we see the two peaks at T and $2T$. When noise is increased up to $D=0.25$ ps, the system response to the modulation is increased by noise, so that we can still ensure that the system in mainly dominated by the modulation although helped by the noise. If noise intensity is further increased, the peak in the PDF moves to lower periods and diminishes in height. This movement towards shorter periods between dropouts is the consequence of the fact that noise dominates the dynamics of the system, as it can also be seen in Fig. 2(c).

Once the effect of noise intensity is understood, we study the effect of the noise correlation time in the system. From Eq. (3), there are two parameters of the noise that can be controlled: the noise intensity D and the correlation time τ_c. From these parameters we obtain the noise amplitude σ. From now on, the value of σ is fixed to 0.035 and τ_c is increased from a few picoseconds to thousands of nanosec-

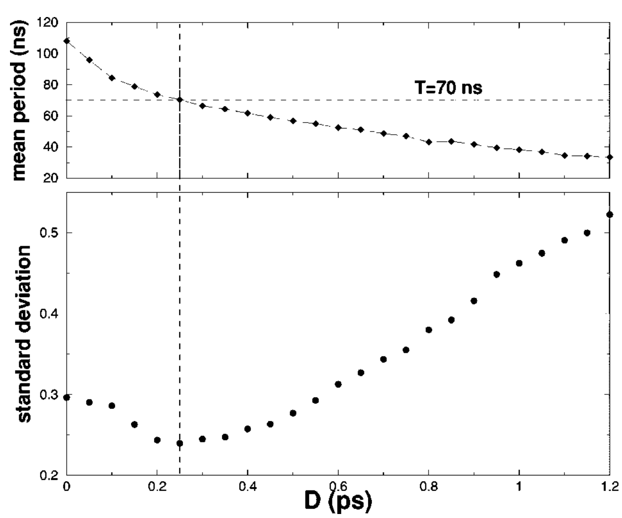

FIG. 3. Mean and relative standard deviation of the dropout periods versus noise intensity. Parameters are those of Fig. 2.

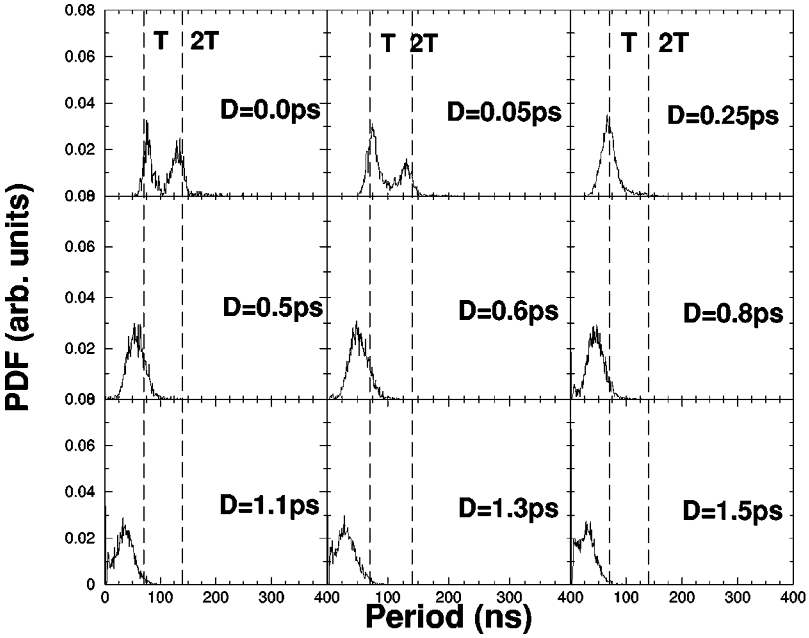

FIG. 4. Probability distribution functions (PDF) for increasing values of noise intensity, for the case of Fig. 3.

onds, in order to study its effect in the dropout regularity. Correlation time of the noise has been demonstrated to be a critical parameter due to the fast time scales of the system [16]. The white noise limit cannot to be reached due to experimental limitations and to the fast dynamics involved in the system, and noise correlation time has to be considered. In Fig. 5 we present the time traces for three different correlation times, together with the corresponding PDFs. The results obtained are qualitatively the same as those found when noise intensity was increased. An improvement of the system response to the modulation is observed when $\tau_c \sim 200$ ps [Fig. 5(b), see also Fig. 6], which allows us to conclude that SR is also observed for an optimal value of the noise correlation time. As in the previous case, the maximum regularity of the system is also achieved when the mean period of the dropouts is equal to the modulation period, as can be seen in Fig. 6. The explanation of this phenomenon is that in semiconductor lasers, carriers play an important role in the dynamics induced by the noise. When high-frequency external noise is added to the system, the carriers cannot follow the fast evolution [see Eq. (1)], and they act as a noise filter. This is the reason why we cannot see any effect for low values of the noise correlation time (see Fig. 5). In fact, as the noise correlation time is decreased, we would need to increase the noise amplitude to have similar effects. On the other hand, for low-frequency noise, which means large correlation time (~ 1 ns), the carriers are able to follow the noise dynamics. It will only be for intermediate values of the noise correlation time, that the system will be assisted by the noise to follow the modulation period.

In conclusion, we have shown how external noise can enhance the effect of a weak modulation signal in a semiconductor laser in the LFF regime, leading to an entrainment of the power dropouts exhibited by the system. This stochastic entrainment can be considered as a particular example of stochastic resonance. We have given examples in which noise clearly assists the harmonic forcing by improving the

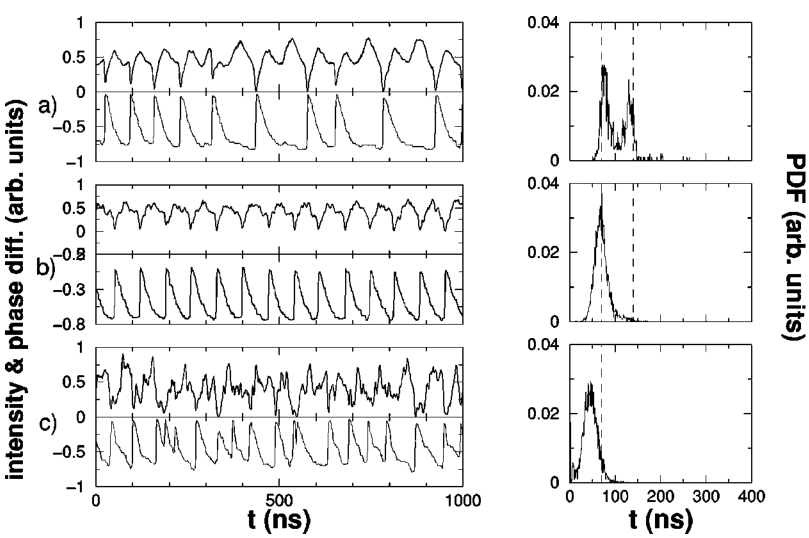

FIG. 5. Time evolution of the laser intensity (left column, upper plots), the phase difference $\eta(t)$ (left column, lower plots), and the corresponding PDFs of the dropout periods (right column) for increasing values of the external-noise correlation time: (a) $\tau_c = 15$ ps, (b) $\tau_c = 200$ ps, (c) $\tau_c = 1920$ ps. The noise amplitude is set to $\sigma = 0.035$, the modulation amplitude to $A = 0.03$, and its period to $T = 70$ ns. Other parameters are those of Fig. 1.

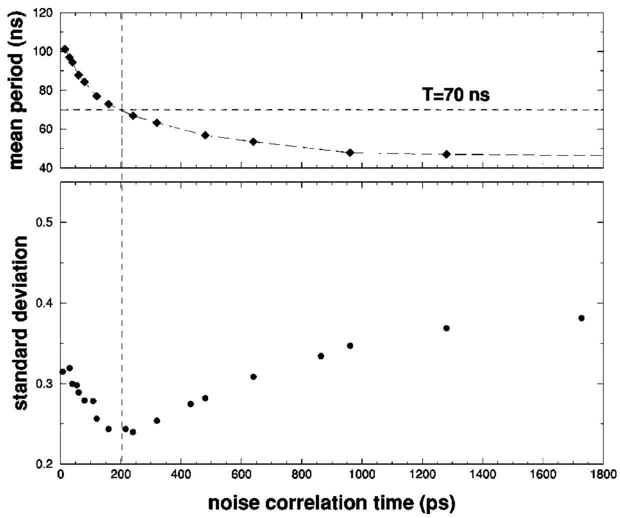

FIG. 6. Mean and relative standard deviation of the dropout periods versus noise correlation time, for the case of Fig. 5.

system response. This external noise cannot be considered white because the dynamics of the system contains subnanosecond time scales smaller than those of the electronics involved in the process of noise generation. In fact, the intensity of a pure white noise having the same effect as the one reported here is unrealistically high. Optimal entrainment can be achieved by controlling the amplitude and/or the correlation time of the noise. Both parameters have a certain range of values where the entrainment effect is maximum. The physical significance of the optimal correlation time is still unclear. These results may motivate experimental investigations about how noise correlation time affects stochastic resonance.

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